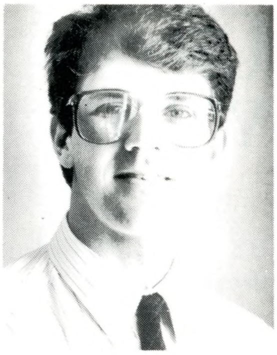
The Geodesy Corner

GPS APPLICATIONS - AMONG OTHER THINGS



BY JAMES FERGUSON, O.L.S.

Well it finally happened. I've been warned against it a thousand times, shunned the advice for all but my *most important tasks*, and said "It can't happen to me!". What is so terrible that it ranks right up there with running out of gas on the 401 just as it turns into 24 lanes, or spending weeks working on a model airplane only to have it get stepped on by the dog the very day of its maiden flight, or perhaps dropping your prescription glasses down the wishing well at Wonderland? Of course I'm talking about a computer nerds' worst nightmare - A CRASHED HARD DISK!

Such was the case with the first version of this edition of the Geodesy Corner - or should I say the proposed edition of the Geodesy Corner. Having typed happily along for about 50% of the article, I took my trusty computer on a field job *without having made recent backups.* As I have alluded to, I lost the complete contents of my hard drive on about day five of the project. All of the recent survey data was backed up (important stuff you see), but the mundane directories for word processing, software packages and my articles were not. Alas, c'est la vie. In any event here I am reworking a piece that had already seen an earlier birth.

Since the last Corner, I received a query which was passed on by the offices of the Association. The question, sent in by Andrew Mantha of Archibald Gray and McKay, brought up the subject of the synchronization of GPS satellite and receiver clocks. More specifically, Andrew's question delves into the effects of the theory of relativity on the GPS signal, and how the satellite and receiver clocks are kept synchronous as the satellite travels at almost 4 km/second in an orbit some twenty thousand kilometres above the earth's surface. Phrasing the question slightly differently, one can ask how the GPS clocks accommodate the fact that the GPS signal is being transmitted at about the speed of light from a high velocity satellite, and are received at a GPS receiver at or near the surface of the earth. The earth based receiver is in a different inertial frame from the satellite, and is moving at a different velocity than the satellite.

Based on the work done by a Dutch physicist named Hendrik Lorentz, a transformation known as the Lorentz Transformation predicts that "the time interval between two events does not have to be the same for observers in relative motion". His transformation takes into account the results of experiments performed by Michelson and Morley in 1881, which proved that the velocity of light was the same in all directions, regardless of the direction of motion of the observer. This was a surprise to the two physicists, and it wasn't until Einstein's famous statement of the principle of relativity in 1905, that the puzzle was solved. Einstein stated that "all laws of nature must be the same for all inertial observers moving with constant velocity relative to each other". In order to arrive at a practical solution for the problem of an object (or signal) moving at the speed of light relative to another observer, the formulation of equations must take into account the Lorentz transformation. This means that our GPS signal emitted at the speed of light from a high velocity satellite, appears to be moving quicker relative to an observer on the earth.

On board each satellite there are several atomic clocks, or frequency standards, used for the precise timing necessary for the GPS system to operate. The emissions from these frequency standards are affected by the two common forms of relativity - special relativity (the satellite's velocity) and general relativity (the difference in gravity potential between the satellite and the receiver) (Leick, 1990). For all satellites, the fundamental frequency for each atomic clock is 10.23 MHz. Using this basic frequency, the satellite generates the various components of the GPS signal as a multiple of the clock frequency. The signal includes the basic carrier wave information of each of the L1 and L2 frequencies, as well as the navigation messages, satellite ephemerides and other information by which the carrier wave is modulated.

In order to compensate for the effects of special relativity (the satellite clock is too fast) the onboard standard of 10.23MHz is set slower at the factory to 10.22999999545 MHz. Thus, when the signals are acquired by a GPS receiver ultimately in a different inertial frame, the clocks will be virtually synchronous. The second effect of relativity, that caused by the difference in gravitational potential between the satellite and receiver, is determined to be proportional to the eccentricity of the satellite's orbit. Thus for a near circular orbit, the correction will be zero. The GPS satellites do have eccentric orbits, and the resultant error manifests itself in terms of a timing error, which can be easily translated into an error in the computed range between the satellite and the receiver. To address this effect, most GPS receivers apply the correction internally, taking into account the orbit eccentricity, the semi-major axis, and one or two other elements of the orbit. In the case of two receivers observing in a static relative mode, the correction would cancel. The same would not be true for two receivers in dynamic motion, especially if they were in different frames of reference (i.e. two aircraft travelling independently).

The question of the effects of relativity on the GPS system has not been fully resolved by the "experts in relativity", and ongoing research is looking at other ways the theory of relativity impacts on the results of GPS. Since the late seventies and early eighties when most of the theoretical aspects of the system were laid out, many new and innovative applications have appeared, and the system has expanded tremendously. Various problems that were not fully anticipated have become evident, and the evolution of the theories that drive the system is ongoing. By no means have we heard the last of the questions that continually face the GPS community.

It was my intention in this edition of the Geodesy Corner to expand on my last column by introducing ways in which the Global Positioning System can be useful to the surveyor on a day-to-day basis. As well, I would like to cover those "special" applications which may or may not have appeared in your survey practice yet. The topics will span a couple of issues, so this article intends to start the discussion. By no means will all possible applications be covered, but I hope to give you some ideas to work with. I prefer to deal here with overall principles, and why the GPS allows the application, rather than focus too much on the application itself. As a surveyor, it is your job to decide how to best apply the technology from a pragmatic point of view.

GPS as an EDM

This may be the simplest way to use reduced GPS data, and yet the distance itself is not a direct observation of the system. We know from the last Geodesy Corner that the basic computations of GPS raw data yield three dimensional cartesian coordinate differences when using a minimum of two receivers in a relative positioning mode. That is, differences in each of cartesian X, Y and Z are calculated between pairs of stations. In order to derive a distance from a single pair of simultaneously receiving GPS stations, one must use the three coordinate differences to arrive at a solution. For this purpose, we do not need an extremely accurate geographic starting point for our survey "system", as long as we know an initial position to about 30 metres in the satellite frame of reference. We arrive at a distance by calculating the square root of the sum of the squares of the three differences. Note that this will be a spatial slope distance between the top of each monument. There are many combinations of differences that will translate into an equivalent distance, and this is where the power of GPS as a distance measurer is evident. Following on this idea, it is not critical that the orientation (the dx, dy and dz values) of the computed baseline be true, in order for the distance to be extremely close to its' true value. We will discover however, that the azimuth GPS delivers is actually quite good. Thus we discover another application for a pair of GPS receivers, the calculation of the orientation, or azimuth of a line.

GPS for Azimuth

As an autonomous system, GPS has the inherent property of providing azimuth between any pair of GPS receivers in the satellite frame of reference. Remember that all GPS computations are referenced to the WGS84 ellipsoid, so that the result of azimuth determination with GPS is a geodetic azimuth within the WGS84 system. Since the GPS constellation is under constant monitoring it is a very stable system, and one could actually think of the entire GPS satellite system as a large space based control network. This being the case, any derivation of azimuth within the system should be repeatable with a certain degree of reliability.

Tests that I have done with GPS derived azimuths over a variety of distances, have concluded that the maximum error one might expect between repeat measurements of the same line is on the 3 or 4 second level. What I also discovered was the minimal change in azimuth when the values were "stand alone" (one line at a time) as compared to the same azimuths being extracted from a proper network adjustment. This shows good promise for azimuth determination using only two receivers when no geodetic control is available. GPS azimuth will also be very close to an actual NAD83 azimuth, due to the fact that this datum uses a reference ellipsoid (GRS80) which is virtually identical to WGS84.

Many surveyors perform work which must be referenced to an astronomic system, and direct substitution of a geodetic GPS azimuth may not seem possible under these circumstances. Geodetic and astronomic azimuths differ by a correction termed the Laplace correction. This difference is computed from the deflection of the vertical (plumb line) which is the angular distance between a line perpendicular to the geoid and one perpendicular to the satellite ellipsoid. Throughout Canada, the Laplace correction varies from one or two seconds up to a maximum of about twenty seconds. Therefore a geodetic azimuth will differ from the astronomic azimuth of the same line by this amount. For legal surveys, a difference of several seconds is not significant, especially if the astronomic azimuth is normally calculated using a sunshot. For a rigorous survey network this would not be acceptable, and proper modelling of the vertical deflections would be necessary to arrive at a good approximation of astronomic azimuth.

Several GPS surveys that we've done have involved the use of sunshots for orienting the survey, and actual differences from geodetic azimuths range from 2 to 30 seconds.

I'll end the article here, and continue on with more GPS applications next issue. Please continue to send your comments, questions and suggestions.

References:

- * Leick, Alfred - GPS Satellite Surveying (1990)
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- * Physics - Alonso - Finn (1972)

James Ferguson graduated from the University of Toronto with a B.Sc. in Survey Science and received his OLS in 1990. As the Vice President of GEOsurv Inc. in Ottawa, Mr. Ferguson applies his extensive experience working with Global Positioning Satellite (GPS) technology to both user and applications development projects. He has been involved in using static and dynamic differential GPS solutions to solve many mapping and control problems. Over the last few years, Mr. Ferguson has devised methods for combining GPS and conventional terrestrial data in integrated adjustments. More recently he has been working on the development of GEOsurv's training unit, including GPS Field Camp^{tm}.